INDEPENDENCE SCHOOL DISTRICT

Inspiring Greatness

Math Virtual Learning

Algebra IIB

Using an Exponential Equation to Solve Compound or Continuous Interest Problems

April 20, 2020

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Lesson: April 20, 2020

Objective/Learning Target: Students will find interest using the Compound Interest Formula or the Continuous Interest Formula

Let's Get Started:

Interest is the amount of money you earn on an investment or the amount of money you pay on a debt.

Examples of investments would be the amount of money you have in a bank account or the amount of money you have in the stock market.

Examples of debts would be student loans, car loans or credit card debt.

Watch the first 10 minutes of this video and fill in the vocabulary guide on the next slide.

Introduction to Compound and Continuous Interest

Compound Interest

Interest is the amount of money you earn on an investment or have to pay on a debt.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Define each variable used in the equations

A: ______________________________

P: ______________________________

r: ______________________________

t: ______________________________

n: ______________________________

Continuous Interest

$$A = Pe^{rt}$$

Compound Interest

Interest is the amount of money you earn on an investment or have to pay on a debt.

rate

time

Amount

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Principal

number of payments in a year

Define each variable used in the equations

A: The amount of money you have at the end

P: The amount of money you start with

r: The interest rate - change to a decimal
Example: 3% is 0.03

t: The time in years

n: The number of payments in a year

More about n

n is the number of payments in a year. In a word problem it is usually identified with a word. Look up how many times per year each of these words represents:

1. Yearly ______
2. Daily ______
3. Monthly ______
4. Semi-annually ______
4. Bi-weekly ______
6. Quarterly ______
5. Semi-monthly ______
8. Weekly ______

Let’s Practice

Example 1: Your 3 year investment of $20,000 received 5.2% interest compounded semi-annually. What is your total return?

A. Define your variables
B. Substitute in the values into your compound interest equation
C. Find the answer by simplifying in your calculator

Example 2: Your 3 year investment of $20,000 received 5.2% interest compounded continuously. What is your total return? Because it says “continuously” use the continuous interest formula.

A. Define your variables
B. Substitute in the values into your compound interest equation
C. Find the answer by simplifying in your calculator

Example 3: Your investment of $20,000 received 5.2% interest compounded monthly. How much time do have to invest it in order for it to reach $30000?

A. Define your variables
B. Substitute in the values into your compound interest equation
C. Because the unknown variable is t, rewrite as a logarithmic equation.
D. Find the answer by simplifying in your calculator

Answers to Practice

Example 1: Your 3 year investment of $20,000 received 5.2% interest compounded semi-annually. What is your total return?

A. Define your variables: t=3, P=20000, r=0.052 n=2
B. Substitute in the values into your compound interest equation
C. Find the answer by simplifying in your calculator: A=$23329.97

$$A = 20000\left(1 + \frac{0.052}{2}\right)^{2\cdot 3}$$

Example 2: Your 5 year investment of $5,000 received 10% interest compounded continuously. What is your total return? Because it says "continuously" use the continuous interest formula.

A. Define your variables: t=5, P=5000, r=0.1
B. Substitute in the values into your compound interest equation
C. Find the answer by simplifying in your calculator: A=$8243.61

$$A = 5000(e)^{0.1(5)}$$

Answers to Practice

Example 3: Your investment of $20,000 received 8.5% interest compounded monthly. How much time do have to invest it in order for it to reach $30000?

A. Define your variables: P=20000, r=0.085, n=12, A=30000
B. Substitute in the values into your compound interest equation
C. Because the unknown variable is t,
rewrite as a logarithmic equation.
D. Find the answer by simplifying in your calculator:
approximately 4.884 years.

$$30000 = 20000\left(1 + \frac{.085}{12}\right)^{12 \cdot t}$$

$$\frac{30000}{20000} = \left(1 + \frac{.085}{12}\right)^{12t}$$

$$1.5 = (1.007)^{12t}$$

$$12t = \log_{1.007}(1.5)$$

$$t = \frac{\log_{1.007}(1.5)}{12} = 4.84$$

Independent Practice

Use the compound interest formula to solve for A.

1. $A = ???, P = \$600.00, r = 4\%, n = 1, t = 10$

2. $A = ???, P = \$1200.00, r = 4\%, n = 1, t = 5$

3. $A = ???, P = \$150.00, r = 3.5\%, n = 4, t = 20$

4. $A = ???, P = \$12{,}550.00, r = 2.8\%, n = 2, t = 2$

5. $A = ???, P = \$7{,}000.00, r = 9\%, n = 12, t = 3$

6. $A = \$1403.60, P = ???, r = 6.8\%, n = 12, t = 5$

Independent Practice

7. An initial deposit of $5,000 is made into a savings account that compounds 7.1 interest annually. How much is the account at the end of the year?

8. After 80 years of 5.8% interest compounded monthly, an account has $102,393.44. What was the original deposit amount?

9. Bank A is offering a 2.7% compounded annually, savings account guaranteed for three years. Bank B is offering a 1.9% compounded monthly, savings account guaranteed for two years. Which bank would yield the most on a principal of $500.00? What is the dollar amount difference between the two bank accounts?

10. How much would need to be deposited into an account earning 4.7%, compounded quarterly, so that the balance would be $1,000,000.00 in 20 years?

Independent Practice

11. Mary discovers a bank account her parents left for her that was opened when she was born 50 years ago. The statement she found states the deposit amount of $100.00 was into an account earning 1.8% compounded quarterly. What is the balance of her account now?

12. In the same box, Mary discovers another statement for an account her grandparents opened for her when she was born. This statement shows a deposit amount of $100.00 made into a 3.6% account, compounded quarterly. How much is in this account now?

13. Luckily, Mary finds a third statement for an account her Aunt opened for her. This was also $100.00 at 1.8%, but it is compounded monthly. How much is in this account now? Based on the answers for each of the accounts Mary discovered, is it better to compound more often or earn a higher interest rate?

Independent Practice

14. A credit card has an initial balance of $550.00. The interest accrues continuously at a rate of 26%. How much will the balance be after 1 year if no payments are made? After 5 years?

15. What was the principal for a continuously compounded account earning 3.9% for 15 years that now has a balance of $2,500,000.00?

16. A student has $34,300 in student loan debt. She defers making a payment for 5 years and interest is calculated monthly. What is going to be her new balance on the student loan?

Answers to Independent Practice

1. $888.15
2. $1459.98
3. $301.14
4. $13267.70
5. $9160.52
6. $1000.00
7. $5355.00
8. $1000.00
9. Bank A; 541.6-519.35= $22.25
10. $392,774.20
11. $245.46
12. $600.11
13. $245.79; higher interest rate
14. $713.31; $2018.11
15. $1,392,764.66
16. $46265.56